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TR.456.

J.W. COX.

Baddow Research Laboratories,
West Hanningfield Road,
Great Baddow,
Chelmsford,
ESSEX.

November, 1942

THE CALCULATION OF OBLIQUE-INCIDENCE TRANSMISSION

Summary

The methods in use for calculating skip frequency and oblique ionospheric transmission are described and compared, and recommendations made as to the best method to use. As a result, improvement in the figures reported by ionospheric observatories to define the ionosphere is suggested.

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Sketches RD/SK 4113
RD/SK 4131
RD/SK 4130
should accompany this report.

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THE CALCULATION OF OBLIQUE-INCIDENCE TRANSMISSION.

§1. Introduction.

For some time past different ionospheric observatories have used different methods of reporting the data they obtain, and of using these data to calculate transmission by way of the ionosphere. This paper reviews the principle methods in use and shows why the revised methods now adopted by the I.S.I.B. at Great Baddow have been chosen.

§2. Scope.

Quite early in the history of short-wave communication several authors treated the oblique-incidence problem sufficiently to develop the main outlines, and these are now well understood and agreed on. The complete calculation is, however, much too complicated arithmetically for practical use, and I am only concerned here with finding a sufficiently approximate model to give useful results, and with discussing convenient ways of quick calculation adapted for routine use. Much of the material is therefore not new, but an examination of existing methods. I have therefore refrained from extensive references to literature, and have not given any of the numerical results obtained except where necessary to bring out a particular point.

Two problems arise which are intimately related but must be separated. Firstly, given the state of the ionosphere as expressed by vertical-incidence methods, to calculate oblique transmission. Secondly, to express the results of measurements in a concise way so that all useful information can be circulated and extracted by the recipient with a minimum of labour, The treatment of the second problem obviously depends on the solution of the first.

The assumptions about the ionosphere which must be made in order to obtain a soluble problem may be first listed. These are:-

(1) The earth is spherical.

(2) The atmosphere is homogeneous and has unit refractive index everywhere below a certain height.

(3) Above this height the atmosphere is ionized and contains free electrons and molecular ions. The number of ions is such that on account of their large mass, as compared with the electrons, their effect is negligible.

(4) The density of electrons per unit volume N is a function only of the height h above the ground and not of position on the earth's surface.

(5) N is a slowly-varying function of h, in that neither N or dN/dh change by a large fraction in a range of h comparable with the vacuum wave-length used. Thus "ray" solutions of what is essentially a wave problem may be used.

(6) The Lorentz "polarization" term is zero.

(7) The effect of the earth's magnetic field is neglected.

(8) The molecular density is so small that the effects of gas-kinetic collisions may be neglected.

(9) No other signals or noise exist.

(10) Regard must be paid to the existing ionosphere in determining whether algebraic approximations are justified.

It is evident that these assumptions do not strictly reproduce the actual ionosphere, but it is agreed that if the problem can be solved within there limits, most of the deviations may be expected to be small, or can be allowed for.

§3. Analysis.

The relevant analysis is contained in a number of papers and need not be repeated. In all cases it is found that the general problem is too complicated, and the solutions used are based in some way on the relations for a plane earth. The important relations are:-

(1) If $p'(f, \alpha)$ is the equivalent path-length (path time devided by velocity of light in a vacuum), for a frequency f at angle of elevation α, then

$$\sin\alpha \cdot p'(f, \alpha) = p'(f \sin\alpha, \pi/2) \quad \text{........(1)}$$

(2) The horizontal range covered before returning to the ground is given by

$$D. \ (f, \alpha) = \cos\alpha \, p'(f, \alpha) \quad \text{..............(2)}$$

These two theorems lead to the further relation

$$D\ (f, \alpha) = \cot\alpha \, p'(f \sin\alpha . \ \pi/2) \quad \text{........(3)}$$

These equations give all the information necessary for transforming the $p'(f, \pi/2)$ curve as normally measured into corresponding curves giving similar information for transmission between separated points, without reference to the actual relation between ionization and height [N (h) curve] other than as expressed in the $p'(f, \pi/2)$ curve. If more than one N(h) curve can give the same $p'(f, \pi/2)$ curve (as may often be the case) the oblique transmission remains the same.

For a given frequency f_{ob} at oblique incidence and angle α, there is an "equivalent frequency" $f_{ob} \sin\alpha$ and the distance traversed is $\cot\alpha \ \ p'(f_{ob} \sin\alpha, \pi/2)$. Moreover if only a limited range of

~~and~~ f_{ob} is required for a special problem, it is sufficient to know the $p'(f, \pi/2)$ only for a corresponding range round $f_{ob}\sin\alpha$, and if the N(h) curve is used in the discussion, any curve that reproduces this particular range will be satisfactory. Care must then evidently be taken not to use this N(h) curve outside the range in which it was chosen to fit.

All these relations are only strictly true if the earth can be considered flat. If the curvature is appreciable, as will be the case if the angle of elevation at the ionosphere is not large compared to the bending of the layer in the distance traversed, or if the horizontal distance traversed in the ionosphere is comparable with or larger than its thickness, the relations are modified and are no longer independent of the N (h) curve. They do, however, remain true to the first order in h/r_o (where r_o is the radius of the earth) and consequently it is possible to find modified solutions in which much of the simplicity of the flat earth relations is retained.

Before passing on to consider these modifications, it must be noted that the practical application of the methods is affected by the duality of the $p'(f, \pi/2)$ curve produced by the geomagnetic field. In all discussions it is assumed that the curve for the "ordinary" ray may be used without effect from the field. This assumption is doubtful though experimentally true at reasonable distances, and is retained as the best available.

I now consider in detail the methods of quick calculation based on these general ideas, as modified to include the effect of curvature. The two methods in use may be named the "transmission curve" and "parabolic" methods, and will be dealt with separately.

§ 4. Transmission Curves.

The original "transmission curve" methods are not at present in use here although they are in America. They may have some future application here, and are described as the easiest approach to the methods actually used.

Suppose that the $p'(f, \pi/2)$ curve is drawn on arbitrary scales of p' and f and that it is desired to transmit obliquely on a frequency f_{ob}. Then any angle α determines a frequency $f_{ob} \sin\alpha$. If the distance D is also given, (3) determines the value which $p'(f_{ob} \sin\alpha, \pi/2)$ must have for transmission to take place. Thus for a given D and f_{ob} there is a "transmission" curve relating $f_{ob} \sin\alpha$ to p', and the intersection of this curve with the experimental $p'(f, \pi/2)$ curve determines the transmission Any point on the transmission curve corresponds to a definite value of α and also by (1) to a definite $p'(f_{ob}, \alpha)$. Thus all circumstances of the transmission are determined.

For a given f_{ob} there are therefore a set of curves for varying D. Similarly for a given D there is a set for varying f. If the $p'(f, \pi/2)$ curve is always on the same (arbitrary) scale, as may be the case with an automatic recording arrangement, sets of such curves may be kept ready prepared. Transmission calculation is then simply a matter of laying the curves over the record. It is not necessary to measure the record itself.

If, however, the records are not all on the same scale they must be measured and redrawn, and a modification of this procedure is used. It can be seen from (3) that if a logarithmic scale for f is used, the curves for a given D will all be the same shape, displaced along the f axis according to log f. Thus it is only necessary to draw one such curve, and it is possible to combine all the information in one set of curves, each for a particular value of D, the whole set being slid along the axis of log f. The scale of h is not important but it is found convenient in practice to make it logarithmic as well. Any point on one of these curves corresponds to a definite value of α or $p'(f, \alpha)$, and these values may be marked on the curves.

This simple theory only applies in detail when, in addition to the restrictions 2 - 10 of paragraph 2, the earth is considered plane. It is however, so easily handled that it seemed useful to try an extension to the actual case of the curved earth. This extension has been made, and it is immediately found that not only the $p'(f, \pi/2)$ curve must be known but also the N(h) curve. Now although the $p'(f, \pi/2)$ curve can be found from the N(h) curve, the reverse process is in many important cases not unambiguous, and the calculation therefore cannot be made exact. However, since it is found that the dependence on the N(h) curve is only of the second-order in h/r_o where r_o is the radius of the earth it has been possible to obtain a modified set of transmission curves which retains all the features of the original set while being more nearly accurate. There is however still some approximation which has to be made in a somewhat arbitrary manner. The approximation lies in assuming approximate values of the true height, and composite curves are obtained to correspond roughly to the probable values. The transmission curves are in fact an approximate algebraic solution of a fairly good physical representation of the complete problem. The technique has also been extended to include the second-order terms but the required knowledge of the N-h curve makes it less readily handled except when applied to region E. In this region the problem is simplified by the smallness of the variation in height, and the curves have been modified to include higher-order terms at 1500 - 2500 kms. with virtual heights less than 160km.

When using these curves for skip calculation they have to be placed tangential to the $p'(f, \pi/2)$ curve, and it has been found useful to record the "skip factor" F which is the ratio of the M.U.F. to the critical frequency. As a result of experience it has been noted that to a degree of accuracy comparable with that attainable in the setting, a given factor for one distance for region F implies a definite factor at any other distance. Thus it is not necessary from this point of view to use the curves for any other than a standard distance. In practice however, it is best to use all the curves so that allowance can be made for control of skip-distance by different regions at different distances.

§5. "Parabolic Layer" method.

This method is also based upon the fundamental ideas of paragraph 3 and is adapted to the ready calculation of the frequency for which a given distance will be at the edge of the skip zone (generally called M.U.F.).

The "transmission" curves of paragraph 4 may cut the $p'(f, \pi/2)$ curve in more than one place, each being the "equivalent frequency" for a particular mode of transmission. At the skip frequency two points come together so that the two curves touch. With the ionosphere as it actually exists, it is observed that this point of tangency is almost always fairly close to the critical frequency, so that only a small portion of the $p'(f, \pi/2)$ curve is really required for M.U.F. calculation. Since, on a plane earth, the (N, h) curve is unimportant, it will be legitimate to use any (N, h) curve which gives the same $p'(f, \pi/2)$ curve near the critical frequency independently of whether it gives the same curve at other frequencies.

In the "parabolic layer" method this is actually done. The N(h) curve is chosen so that the integrals occurring in the transmission theory can be solved algebraically as nearly as is necessary. Although the independence of transmission from the N(h) curve is only rigorously true for a plane earth, it is assumed that sufficient allowance for the curvature is made by using rigorous theory on the chosen N(h) curve. The algebraic approximation involved in the transmission curves is thus avoided, and replaced by a physical approximation for the true height, which however, has to be known much more accurately for the parabola than for the transmission curve method. The parabola method is therefore a rigorous algebraic treatment of a rather approximate physical picture.

The details are therefore these. The ionosphere chosen has zero density to a height h_o. At a height Z above h_o, the N(Z) curve is of the form

$$N = N_c\left(\frac{2Z}{y_m} - \frac{Z^2}{{y_m}^2}\right) \quad (4)$$

where N_c is the value of N corresponding to the observed critical frequency f_c.

The $p'(f, \pi/2)$ curve of such a region is given by

$$p'(f\ \pi/2) = 2h_o + y_m\ x \tanh^{-1}x. \quad (5)$$

where $x = f/f_c$.

The function $\varphi(x) = x \tanh^{-1}x$. is tabulated, and thus

$$p'(x, \pi/2) = 2h_o + y_m \varphi(x). \quad (6)$$

Thus if $p'(f, \pi/2)$ is plotted as a function of $\varphi(x)$, a straight line will result, from which h_o and y_m are obtained. It is found in practice that when this is done with the $p'(f, \pi/2)$ curves actually obtained, a reasonably good line is usually obtained over as much range in x as appears to be necessary to include the 'equivalent frequencies' required for M.U.F. calculation.

Having thus obtained the parameters f_c, h_o and y_m of the layer, it is a matter of algebra to obtain the M.U.F. f_s for any distance, the relation (4) being chosen to make this integral tractable. In practice curves are given of $F = f_s/f_c$ in terms of the derived parameters $y_m + h_o$ and y_m/h_o.

It is evident that this method can be formally extended to any distance, so that once the approximation involved in the choice of the type of layer is made the whole process is numerically exact. It is however, not obvious that the error introduced by the choice is in fact small. Special cases of more complicated regions have been worked out both analytically and by this approximate method, and the agreement is reasonably good, but it is not certain that these other assumed layers sufficiently well represent the actual state of the ionosphere. For calculations of transmission other than M.U.F. the method is not applicable since the artificial $p'(f. \pi/2)$ curve may depart greatly from the true one if the equivalent frequency is well below the skip frequency. It is of course possible to fit other parabolic distribution to reproduce parts of the $p'(f\ \pi/2)$ curve other than those near the critical frequency, but there is no a priori knowledge of which is the appropriate part to fit, and in any case a simple parabola cannot deal with parts where dp'/df is negative.

I have here described the method as given by its authors, In practical applications I have found it convenient to alter the procedure slightly. The experimental curves are always drawn at Baddow on a logarithmic frequency scale, and it is therefore convenient to have a transparency ruled with parallel lines at a spacing corresponding to $\varphi(x) = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0$ and ∞. This aids the quick determination of the appropriate values of p'. The line found by plotting p' against $\varphi(x)$ determines $h_m = y_m + h_o$ immediately by its intersection with $\varphi = 1$, and the difference between this and the intersection with $\varphi = 2$ gives y_m directly. The original parameter y_m/h_o although convenient in handling the arithmetic when working out the curves is a derived quantity and I have found it useful to redraw the curves in terms of h_m and y_m as shown in Fig.1. for a distance of 2500 km. (The gap in the figure corresponds to values of y_m/h_o not shown in the originals).

In the original, values of skip-factor F are given for certain distances in terms of the layer parameters, and a separate set of graphs must be used for each distance. Inspection of these graphs however, reveals that if the factor at one distance is known, any corresponding set of parameters will give an identical factor within 1- 2% at any other distance. I have not attempted to obtain analytic confirmation of this, but it is very useful as it means that the set of curves in Fig.2. which are obtained by replotting the originals give all the information needed in practice. In order to obtain a useful range of values it is convenient to use the factor for 2500 km. and the curves of Fig.2 are drawn for convenient values of this number. It must be emphasized that on account of the possible inaccuracies mentioned above I do not necessarily believe that these curves are justified at great distances.

§6. Other Methods.

In passing, it is perhaps necessary to mention that other methods have been used in the past. The earliest that was based on more than a guess at the height of the ionosphere used the theory of reflection from a thin layer, which was assumed, for want of anything better, to be located at a height corresponding to the lowest virtual height. There is no doubt that this method, although useful at one time, can be quite misleading and is now outmoded.

It has also been suggested that the virtual height at 5/6 the critical frequency should be used. This is based on the parabolic layer theory given in § 5 but is evidently misleading, partly beacuse it is incomplete and partly because 5/6 is so far below the critical frequency that it is badly affected by the presence of lower layers.

There is no need to give further consideration to these methods.

§ 7. Comparison of the Two Methods.

When comparison came to be made of the methods of § 4 and § 5 it appeared at first that the results obtained were significantly different. Further investigation shows that the significant differences occur mostly at very great distances. In view of the fact that the basic assumptions begin to go badly wrong at very great distances, especially assumption (4) and perhaps (2) and there is not sufficient available information to enable a reliable estimate of their effect to be made, it is not worth while to quibble about quite a large discrepancy at this distance. In long distance transmission the absolute limiting frequency is of importance, and the limit is partly set by the ground-absorption near the aerials, which limits the effective angle of elevation. It is however, easy to obtain a relation between angle of elevation and skip-frequency for a given N(h) curve, although not easy to find the distance to which this limiting frequency will go. There are found to be significant differences in this limiting frequency for the different N(h) relations which could give the same $p'(f, \pi/2)$ curve and it might be thought possible to differentiate between them by this means. In practice however, the other uncertainties are, at present at least, too large to permit the test to be made. This fact however, shows that each of the curves of Fig.2 should split into several at the greater distances, corresponding to the differing layers with the same $p'(f, \pi/2)$ curve. As, however, there are so many other sources of error, they may be used with caution as they stand, although the bulk of the empirical evidence at present available suggests that the long-distance values may be somewhat low. Thus only a rough value for the limiting frequency can be hoped for.

Leaving the long distances, it was found that there were still discrepancies at the shorter distances. The investigation of these showed incidentally that it is necessary to be very precise about the comparison. It is by no means sufficient to assume that curves taken at Slough and at Baddow at "nearly" the same time will be identical. The differences revealed in such a comparison need not be detailed here, but they are of importance in that short period fluctuations in the ionosphere evidently limit the accuracy that is worth striving for.

When this had been cleared up, there was still a slight residual discrepancy at 2500 km. in that the transmission curves gave consistently a slightly higher factor by about 3 - 4%.

This was unexpected as it was now realized that the two methods are essentially the same. It was traced eventually to the approximations used. In drawing the transmission curves as modified for curvature, allowance has to be made for an estimated value of true height, and this was done in an arbitrary way, joining up graphs by hand in the knowledge that there would not be more than a few percent error, which was not considered to be important in view of the large variations encountered in the ionosphere.

It therefore appeared that the approximation adopted did not give quite the result that could be analytically obtained if the ionosphere were actually of the type in (4) and although the assumption of (4) may involve quite as much approximation physically as the other does algebraically, it was agreed to be desirable that the same result should be obtained on the only analytic solution. The transmission curves have thereforebeen redrawn with the new approximation. A comparison of the two methods carried out on all the curves obtained hourly at Baddow during 1942, October 1 - 7, inclusive is shown in Fig.3. which gives the factor for 2500 km. and it is evident that the agreement between the two is now as good as is justified by the accuracy of the measurements. There is therefore no possibility of preferring one method to the other on the grounds of accuracy. The decision as to which method to use can only be made in terms of convenience, and here it is almost impossible to avoid prejudice in favour of the method with which one is most familiar.

In making recommendations it has to be remembered that the scheme adopted must be suitable for routine use by semi-skilled personnel and that as little loophole as possible should be left for misinterpretation or error. Further, it is desirable that all ionosphere problems should be tackled by a unified technique.

While attention is totally confined to M.U.F. calculations the parabolic method has some advantages. It does not need much of the recorded curve to be measured, although the part used must be measured very accurately. The further calculation is straightforward and does not require any very special tools other than ordinary linear graph paper of any arbitrary scale. It gives two quantities denoted by h_m and y_m which may be tabulated, and which together with the critical frequency contain all the information. Before accepting these however, it is necessary to enquire into their physical significance. It is very easy for those unacquainted with the theoretical foundations to assume that h_m is actually the true height of maximum density, whereas in fact h_m and y_m must be regarded as parameters which serve to determine the shape of a part of the $p'(f, \pi/2)$ curve. The very fact that only a part is determined shows that the physical significance is doubtful and although trial calculations on more complicated layers suggest that h_m is often not much different from the true value, there is no significance in its exact value.

A few trials on practical curves will show that the physical significance of y_m is also doubtful, as it varies enormously and rapidly from minute to minute. At the best it defines the curvature of the nose of the N(h) curve, a quantity evidnetly easily susceptible of wide change by very small fluctuations in N. Although it may be theoretically possible to determine the "scale height" of the ionosphere from y_m, the large fluctuations that occur show that it can only give a rather doubtful statistical measure. However, h_m and y_m together define a factor F at any distance which, as has already been seen, is in itself sufficient for all practical purposes. Moreover, F is not very sensitive to changes in y_m.

From the Laboratory point of view, therefore, it does not matter whether we have h_m and y_m reported or only F for a given distance except that there may be a p ference for using one number instead of two. For routine use however, I would prefer F, because its adoption will prevent those who are not technically qualified from attaching undue importance to the value of h_m. If, having F, an estimate of h_m is wanted, the curves show that for a very large range of y_m such as is encountered in practice, h_m can be obtained within ± 10 km. which is certainly better than its physical meaning.

The transmission curve method gives F directly. Its disadvantage is that it is necessary to plot the $p'(f, \pi/2)$ curve on the scale for which the curves have been drawn, thereby requiring special paper. In the hands of semi-skilled personnel there is however, an advantage at times when more than one ionized region is present. At such times it is necessary on the parabolic method to measure and calculate each layer separately, to see whether the lower layers have the controlling influence at any distance. The transmission curves show this directly, as they must always be placed so that the point of tengency is lower than any cut, and there is therefore less likelihood that control by lower layers will be overlooked. This is particularly true at times when there is a marked F_1 bump but no definite critical frequency for F_1, so that there is scope for misinterpretation in determining what frequency to use. The parabola obtained depends on the value used, and so admittedly does the factor, but if the transmission curves are used there is no uncertainty about the M.U.F.

It may be pointed out that disturbed ionospheric conditions may produce different types of error with the two methods. When the upper end of the $p'(f, \pi/2)$ curve is lost through absorption, the parabola must be referred to a wrong critical frequency and is therefore suspect, whereas the slider is unaffected, unless so much is lost that there is no tangent point. Moreover, on some curves it is found that only a very small portion will give a reasonable straight line for the parabola, and evidently, if account is not taken of all frequencies down to the "equivalent frequency", errors may be produced. On the other hand the transmission curves are more liable to error where the ordinary ray is partially obscured by the extraordinary, and this is more liable to happen when the resolving power of the apparatus used is comparatively poor, as is the case with the automatic gear at present under devolopment. (It may be remarked that automatic gear must almost inevitably have less resolving power than comparable hand-operated gear). There are also of course many other types of ionospheric irregularity equally difficult to interpret for either process.

My personal preference for transmission-curve methods is therefore based on other considerations than M.U.F. calculation and familiarity. A number of ionospheric stations are to be set up with hastily trained personnel, and they will have for operational purposes, to calculate not only M.U.F. but also expected echo patterns at oblique incidence and in some cases angles of transmission. This cannot be conveniently done with the information obtained from the parabola method as it involves the whole of the $p'(f, \pi/2)$ curve.

Approximately drawn sets of transmission curves will solve all these problems directly, by an easily understandable extension of the methods used for M.U.F.

Thus, although the parabola method could be used for M.U.F. and a differing method used to solve other problems the advantages of a unified technique, especially in the hands of routine operators will be lost.

I do not want to describe these other problems in detail here, but the fact that they exist, and are of such importance as to be of great influence in determining the setting up of the new stations, seems to me a quite decisive factor in deciding which method to use.

§8. Method of Reporting.

Having therefore decided that the transmission-curve method has considerable advantages for routine use, the question of reporting must be settled. The ideal would of course be a complete interchange of detailed p'f curves, but as this is not possible, a compromise must be sought. From what has been said already it is evident that all that need be reported are the critical frequencies and some means of expressing the shape of the curves. This is at present done by the American and Australian observatories by giving the minimum vertical height and the height at 0.834 of the critical frequency. With this information it is possible to sketch the shape of the p'f curve and use the transmission slider on this reconstituted curve. Such a method is good so far as it goes, but suffers from the disadvantage mentioned in connection with the parabola method, that it tabulates vertical heights, which in the hands of semi-skilled users may be badly misinterpreted. Moreover, it requires two three-figure numbers to define the shape of the curve. A report equally as useful can be made by quoting the factor for a standard distance. If the curve-shape is required it can be deduced from the factor within the same accuracy as is obtained from the vertical heights. Thus the factor gives as much information, as will save space in transmission, and is less liable to misinterpretation. It is also considerably easier to obtain and tabulate from the original records than the other data as it occurs immediately in routine scaling. For these reasons, therefore, this method of reporting has been adopted at I.S.I.B.

§9. The Magnetic Field.

In most cases the relaxation of the conditions imposed in §2 cannot be done accurately. In particular, this is true of questions of ionospheric horizontal gradient and of signal intensity and scatter. This is not the place to consider how allowance should be made for such deviations from the ideal. In certain cases, however, allowance can be made for the effect of the geomagnetic field, at least to the accuracy desired.

For a plane ionosphere which othersise obeys the restriction of §2, a simple and convenient solution can be given for the M.U.F. in terms of the angle of entry into the layer and of the critical frequency in the cases where the field is either (a) vertical or (b) horizontal, or when, (c) transmission is perpendicular to the magnetic meridian independently of the magnetic dip. The result may be expressed as a correction to the M.U.F. obtained without considering the field, and cases (a) and (c) are found to be identical. The correction is in almost all practical cases small, and falls of rapidly with decrease of vertical angle, so thatit is sufficient to use the approximate relation F = cosec α and to take over the plane earth formulae for the actual curved earth. The identity of cases (a) and (c) mean that in many important parts of the world where the field is nearly vertical a simple correction can be applied. When using the parabola method it is convenient to have graphs showing the relation of f_c to the corrected M.U.F. directly as a function of F, and a convenient method of correctuon has been added to the transmission curve slider. Similar correction can be made for transmission in the meridian near the equator. Other cases cannot be dealt with directly, but it is certain that the corrections required will be of the same order of magnetude as those which can be rigourously obtained, and thus of comparatively little importance apart from short-distance transmission.

§ 10 <u>Conclusions:-</u>

My conclusions are therefore as follows:-

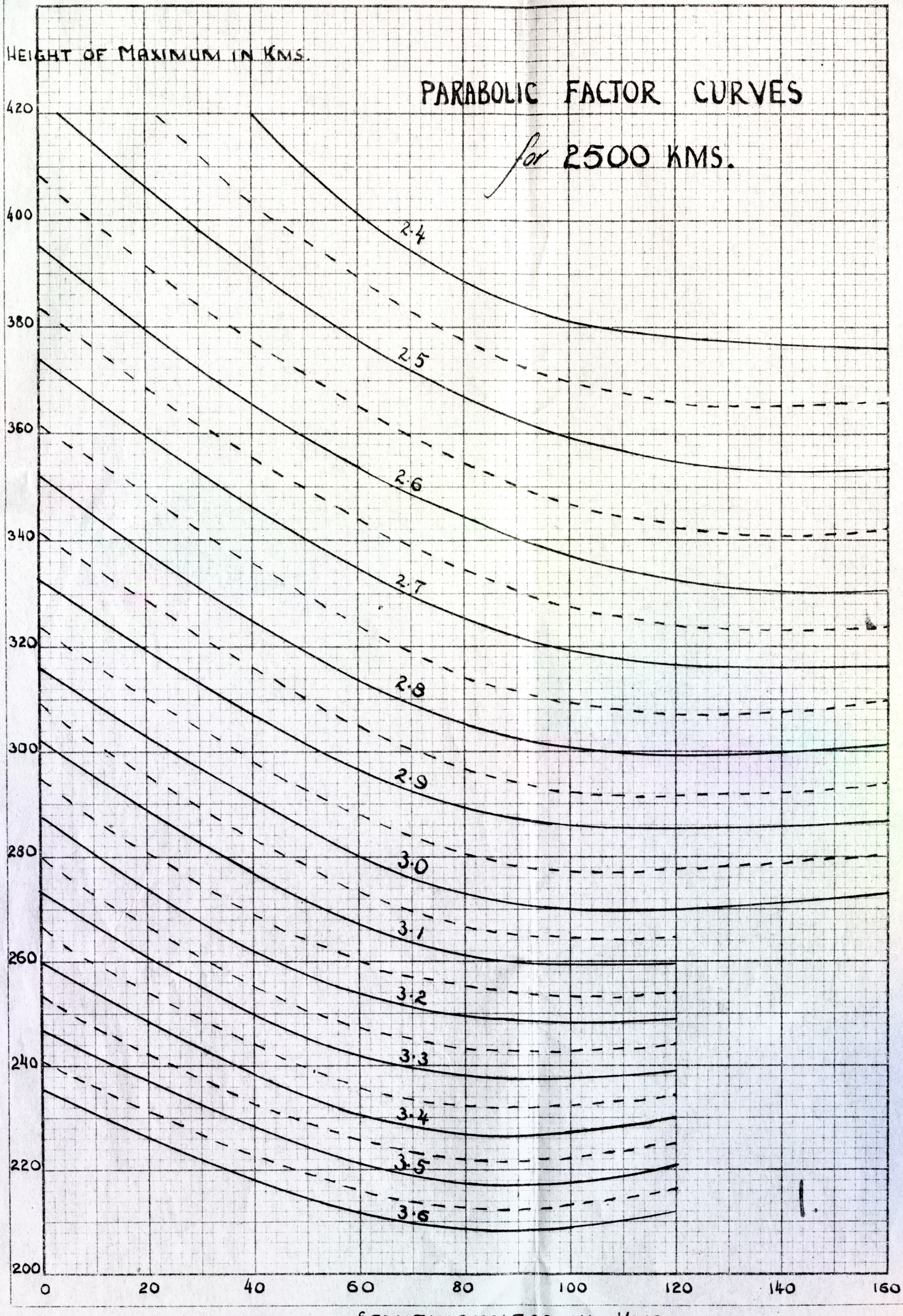
(1) Very long distance transmission cannot be tackled rigorously by any available simple method. The limit is reached about 3000 km, and I would prefer to say 2500 km.

(2) For calculating M.U.F. either of the available simple method is equally accurable, but no physical significance must be given to the numbers used as intermediaries in the parabola method.

(3) For routine use the transmission curve method is the simpler.

(4) For reporting on the ionosphere, the critical frequencies and skip-factors give as much information as any other numbers. This type of report is least open to misinterpretation.

JWC/OML.


HEIGHT OF MAXIMUM IN KMS.
PARABOLIC FACTOR CURVES
for 2500 KMS.
420
400
380
360
340
320
300
280
260
240
220
200
2·4
2·5
2·6
2·7
2·8
2·9
3·0
3·1
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20
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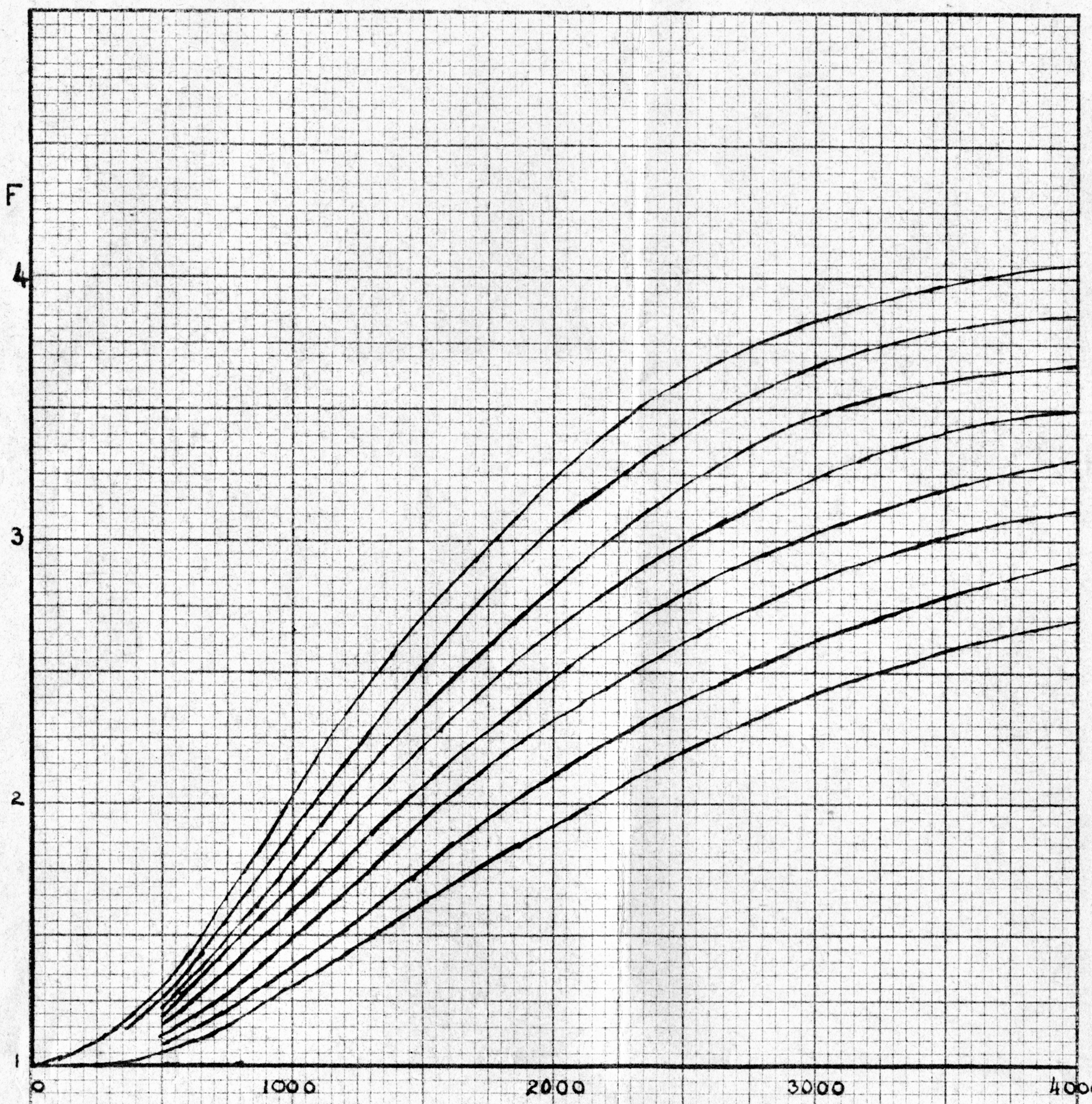
120
140
160
SEMITHICKNESS IN KMS.
1.

RD/SK 4113.



F-d CURVES FOR PARABOLIC LAYER

2.

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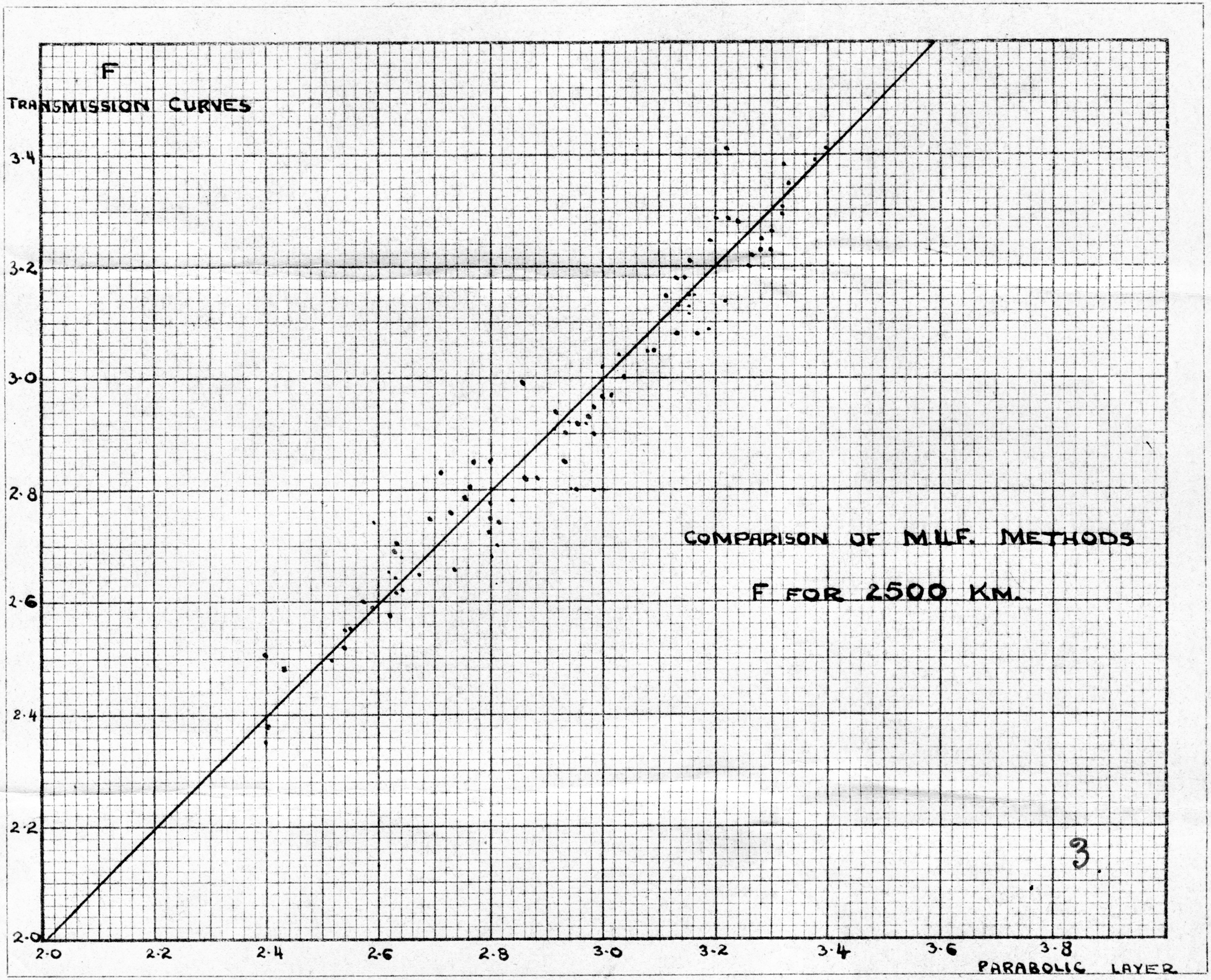

F
TRANSMISSION CURVES
3·4
3·2
3·0
2·8
2·6
2·4
2·2
2·0
2·0
2·2
2·4
2·6
2·8
3·0
3·2
3·4
3·6
3·8
PARABOLIC LAYER
COMPARISON OF M.U.F. METHODS
F FOR 2500 KM.
3.

RD/SK 4130